RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

M.A./M.Sc. FIRST SEMESTER EXAMINATION, DECEMBER 2024

FIRST YEAR [BATCH 2024-26]

COMPUTER SCIENCE

Paper : CC 2

Date : 17/12/2024

Time : 11.00 am – 1.00 pm

Full Marks : 50

Answer any five questions: [5×10]

1. Derive Poisson distribution from binomial distribution.

2. Prove that, when $\rho = -1$, two random variables are perfectly linearly related with negative slope.

3. Using the simple linear regression model $y = \alpha + \beta x + \varepsilon$, prove that (after deriving the OLS estimator $\hat{\beta}$ of β) $\hat{\beta}$ is an unbiased estimator of β.

4. Show, using an example, that cumulative distribution function is right continuous.

5. Consider the following transition probability matrix concerning two-state markov chain about the weather conditions — '0' if it rains, '1' of it does not :

$$\begin{vmatrix} 0.8 & .2 \\ 0.3 & 0.7 \end{vmatrix}$$

Find the probability that it will not rain day after tomorrow given that it is raining today.

6. a) If $H = P + iQ$ be a skew Hermitian matrix, then prove that

   i) P is a real skew symmetric matrix and Q is real symmetric matrix.

   ii) the diagonal elements of H are purely imaginary number or zero.

   b) Let the square matrix A =

$$\begin{vmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{vmatrix}$$

   i) Find all the eigen values of A.

   ii) Is A diagonalizable? If yes, find P such that $D = P^{-1}AP$ is diagonal. [3+7]

7. a) Find the change-of-basis matrix from the basis P of R^3 to usual basis S of R^3, where $P = \{(1,1,0),(0,1,1),(1,2,2)\}$.

   b) Determine the rank and the signature of the following matrix

$$\begin{vmatrix} 2 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & 6 & 10 \end{vmatrix}$$

[5+5]

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